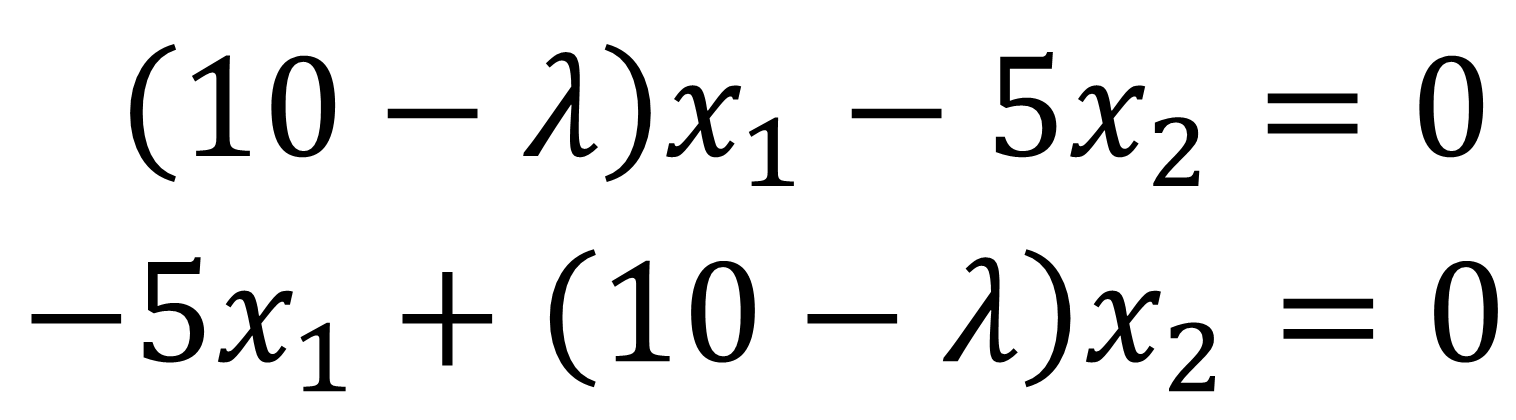
**Practical 9:**

**Objective**: Finding Eigenvalues and Eigenvectors

**Part A:** Use MATLAB to determine the eigenvalues and eigenvectors for the following system



**octave:1>** A = [10 -5; -5 10]

A =

10 -5

-5 10

**octave:2>** e = eig(A)

e =

5

15

**octave:3>** [v, lamda] = eig(A)

v =

-0.70711 -0.70711

-0.70711 0.70711

lamda =

Diagonal Matrix

5 0

0 15

**octave:4>** A = [0.8 0.3; 0.2 0.7]

A =

0.80000 0.30000

0.20000 0.70000

**octave:5>** e = eig(A)

e =

1.00000

0.50000

**octave:6>** [v, lamda] = eig(A)

v =

0.83205 -0.70711

0.55470 0.70711

lamda =

Diagonal Matrix

1.00000 0

0 0.50000

**Part B:**

1. Use MATLAB to determine all the eigenvalues and eigenvectors for the following system

**octave:7>** A = [1 -1 0; -1 2 -1; 0 -1 1]

A =

1 -1 0

-1 2 -1

0 -1 1

**octave:8>** e = eig(A)

e =

3.9251e-17

1.0000e+00

3.0000e+00

**octave:9>** [v, lamda] = eig(A)

v =

-5.7735e-01 -7.0711e-01 4.0825e-01

-5.7735e-01 9.7145e-17 -8.1650e-01

-5.7735e-01 7.0711e-01 4.0825e-01

lamda =

Diagonal Matrix

9.9966e-17 0 0

0 1.0000e+00 0

0 0 3.0000e+00

1. What is the dominant eigenvalue of the above system?

3.0e+00

1. Write the coefficient matrix with dominant eigenvalue in Part (ii).

**octave:10>** lamda = 3

lamda = 3

**octave:11>** [1-lamda -1 0; -1 2-lamda -1; 0 -1 1-lamda]

ans =

-2 -1 0

-1 -1 -1

0 -1 -2

1. Perform row operations to obtain rref of the matrix in Part (iii).

**octave:12>** D = [1-lamda -1 0; -1 2-lamda -1; 0 -1 1-lamda]

D =

-2 -1 0

-1 -1 -1

0 -1 -2

**octave:13>** R = rref(D)

R =

1 0 -1

0 1 2

0 0 0

1. Use manual calculation to find the dominant eigenvector that associated with the dominant eigenvalue in Part (ii).
2. Use MATLAB to verify your answer in Part (v).

**octave:20>** vpower =[1; -2; 1]

vpower =

1

-2

1

**octave:21>** vMATLAB = vpower / norm(vpower)

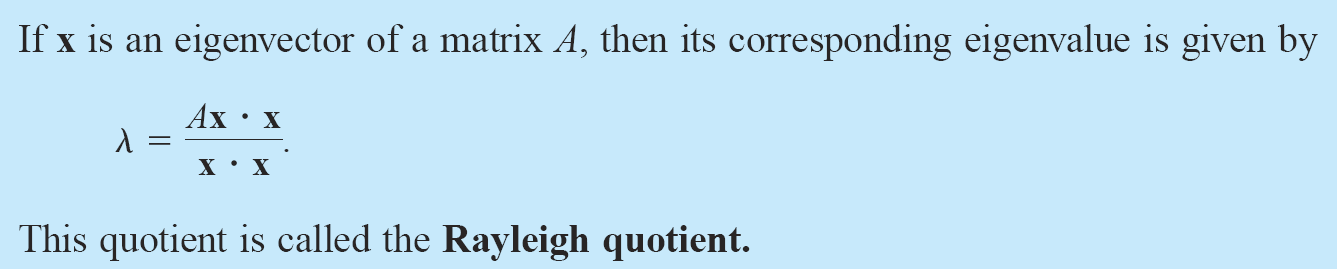
vMATLAB =

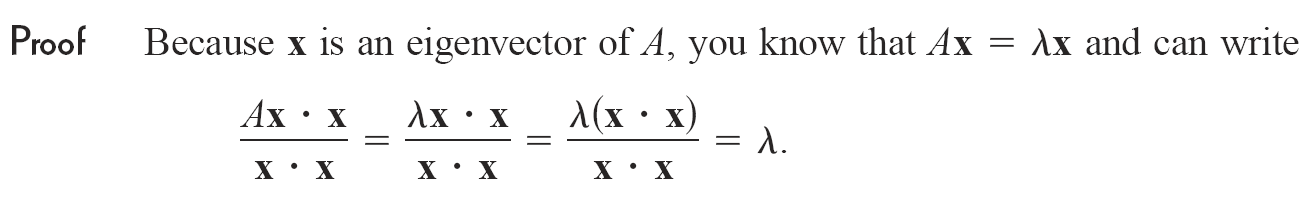
0.40825

-0.81650

0.40825

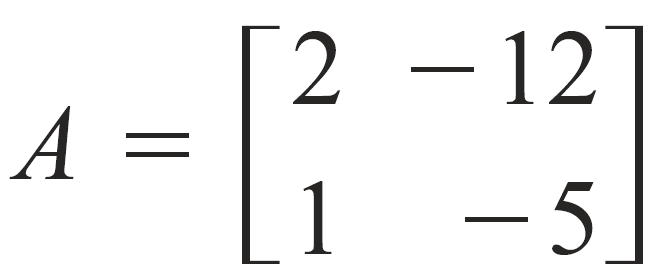
**Part C: Rayleigh Quotient**

****

****

In cases for which the power method generates a good approximation of a dominant eigenvector, the Rayleigh quotient provides a correspondingly good approximation of the dominant eigenvalue.

1. Complete six iterations of the power method (using scaled approximation) to approximate a dominant eigenvector of



**octave:28>** A = [2 -12; 1 -5]

A =

2 -12

1 -5

**octave:29>** x0 = [1; 1]

x0 =

1

1

**octave:30>** X1 = A\*X0

X1 =

-10

-4

**octave:31>** X2 = A\*X1

X2 =

28

10

**octave:32>** X3 = A\*X2

X3 =

-64

-22

**octave:33>** X4 = A\*X3

X4 =

136

46

**octave:34>** X5 = A\*X4

X5 =

-280

-94

**octave:35>** X6 = A\*X5

X6 =

568

190

The approximated X = [2.98947; 1]

1. Use the above result to approximate the dominant eigenvalue of the matrix .

**octave:4>** (A\*X)'\*X

ans = -20.010

**octave:5>** axx = (A\*X)'\*X

axx = -20.010

**octave:6>** xx = X'\*X

xx = 9.9401

**octave:7>** lamda = axx / xx

lamda = -2.0130

**octave:40>** eig(A)

ans =

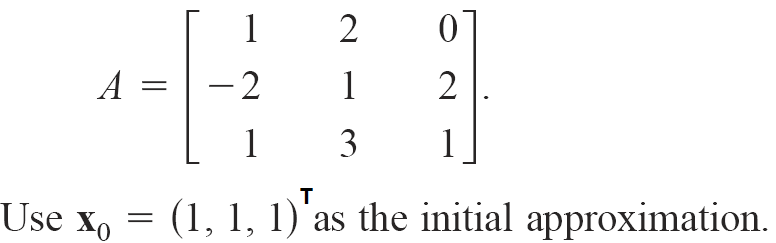
-1

-2

**Part D: The Power Method with Scaling**

The power method may generate vectors with large components. To avoid this problem, we can at each step multiply the vector by

1. Calculate seven iterations of the power method with scaling to approximate a dominant eigenvector of the matrix

****

1. Use your answer in Part (i) to approximate the dominant eigenvalue.

**Part E: Programming Practice**

Write a program to perform operations in Part C. Your program should take the following inputs from the user:

1. Coefficient Matrix,
2. Initial approximation,
3. Iteration size,